EECS 861
Homework 12

1. A received signal X(t) contains pulses of amplitude +1 with width 10 ms plus bandlimited white Gaussian noise N(t). The noise PSD is

$$S_N(f) = \begin{cases} 2.5x10^{-3} & |f| < 500\text{Hz} \\ 0 & elsewhere \end{cases}$$

X(t) is sampled at a rate of 1000 samples/sec. Samples are collected in pulse time sync with the pulses.

a) Design a pulse detector.
b) For your pulse detector and given the parameters above calculate the probability of detection and false alarm.

c) How many pulses are in this record? Describe the approach you used to arrive at that number. This file contains a record of collected samples.
http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/ Find_pulses-2017-a.csv

d) How many pulses are in this record? Describe the approach you used to arrive at that number. This file contains a record of collected samples.
http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/ Find_pulses-2017-b.csv

2. Let $f_X(x;\theta) = \frac{1}{\sqrt{10\pi}} e^{\frac{-(x-\theta)^2}{10}}$. Given $x_1 \ldots x_N$ be N statistically independent samples from $f_X(x)$ Is $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ a biased estimator for θ; yes or no and justify.

3. We want to estimate a received signal X from K observations Y where is modeled as Y=X+N. Here K = 25 and the $\bar{y} = \frac{1}{25}\sum_{i=1}^{25} y_i = 16$.

N is Gaussian with E[N]=0 and Var[N] = σ_N^2

X is Gaussian with E[X]=10 and Var[X] = σ_X^2

N and X are statistically independent. For the following 3 cases:

Case 1: $\sigma_N^2 = 0.10$ $\sigma_X^2 = 10$

Case 2: $\sigma_N^2 = 5$ $\sigma_X^2 = 5$

Case 3: $\sigma_N^2 = 10$ $\sigma_X^2 = 0.10$

Find

a. The MAP estimator for X.
b. The Mean Square (MS) estimator for X.
c. The Maximum Likelihood (ML) estimator for X.

4. Find the unconstrained Weiner filter to estimate S(t) from Y(t) =S(t)+N(t), where N(t) and S(t) are statistically independent and

$$S_S(f) = \frac{8}{1+(8\pi f)^2}$$

and

$$S_N(f) = 1$$

5. Let the desired signal, S[k], be characterized by a moving average process given by S[k]= X[k] + 0.75X[k-1] +0.75 X[k-2] where X[k] is a white Gaussian random process with zero mean and variance = 0.1
   The observed signal is Y[k]=S[k] + N[k] where N(k) is Gaussian with $R_{NN}[n]=0.5$ for n=0 and $R_{NN}[n]=0$ elsewhere.
   a. Find $R_{SS}[k]$.
   b. Find the optimum realizable Wiener filter h[k] where

      $$\hat{S}[n] = \sum_{k=0}^{\infty} h[k]Y[n-k]$$

   c. Apply the optimum realizable Wiener filter h[k] to this received signal http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/received_signal.csv
   d. Calculate the resulting mean square error given the desired signal http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/desired_signal.csv
   e. Repeat parts c and d using h[k] = 1/3, 1/3, 1/3, that is a three sample average and compare the resulting mean square errors.
      (Note: The desired signal and the processed signal needs to be aligned to calculate the mean square error, i.e., there is a delay going through h[k])

6. Let $s_0[k]$ = -2.0, -2.0, -2.0 for k =0, 1, 2 & $s_0(k)$=0 elsewhere and $s_1[k]$ = - $s_0[k]$
   Assume
   $P(s_1) = 0.5 = P(s_0)$
   Y[k] = S[k] + N[k] for k =0…2 where
   S[k] & N[k] are statistically independent
   N[k] is white Gaussian noise with a zero mean and unit variance, i.e., σ_N=1.

   a. Find the MAP decision algorithm.
   b. Find the probability of error.
   c. Apply the MAP decision algorithm for the follow observations
      y(k) = -0.4, 0.1, 0.1 for k =0…2
   d. Repeat a)-c) for $s_0[k]$ = -1.0, -3.175, -1.0

7. A decision is based on 2 samples, Y_1 and Y_2. $Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ is a multivariate Gaussian random vector with $E[Y \mid H_0] = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ and $E[Y \mid H_1] = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ with $\Sigma = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

[Hint see Homework 4 Problem 8]

a) Design the optimum detector.
b) Find P_e.